EFFECT OF THE WEIGHT OF THE ADMIXTURE ON THE TURBULENCE STRUCTURE OF A TWO-PHASE JET

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The effect of gravity on the turbulence structure of an inclined two-phase jet is evaluated according to the Prandtl theory of mixing length.

Earlier studies [1, 2] dealt with the effect of the admixture and the flow unbalance on the structure of a turbulent two-phase jet on the basis of the Prandtl theory, without accounting for the particle weight. In the case of rather large particles and a high specific gravity, the acceleration due to the difference between the particle velocity and the velocity of the gas can become comparable with the acceleration due to gravity. Therefore, the structure of turbulent flow will be affected not only by the "weightless" admixture but also by the particle weight.

In this study an attempt will be made to evaluate the effect of the particle weight on the magnitude of the fluctuation components of the two velocities, gas and particles, and to extend the earlier established relations [1, 2] to the case of a "ponderable" admixture.

When the jet is oriented vertically upward, then the force of gravity decreases the positive fluctuation components of the longitudinal velocity of particles and increases the absolute magnitude of its negative components. As to the gas, conversely, the positive components increase and the negative components decrease. Consequently, as before [2], some asymmetry must emerge in the absolute magnitudes of the fluctuation components of both longitudinal velocities (gas and particles). The transverse component of the fluctuation velocity of particles remains unaffected by the force of gravity.

When the jet is oriented horizontally, then the force of gravity affects the transverse components of the fluctuation velocity and not its longitudinal components. Obviously, however, the shearing stresses have the same magnitude here as in a vertically discharging jet. In the case of an inclined jet the particle weight will affect both components of the fluctuation velocity and must be taken into account in the corresponding equation of motion for particles.

In Stokes approximation we have the equation of motion for particles

$$\frac{dV'^{\pm}_{pi}}{dt} = \frac{18\mu_g}{\rho_s D_p^2}\left(V'^{\pm}_{gi} - V'^{\pm}_{pi}\right) - g_i. \tag{1}$$

Let us express the fluctuation components of both velocities, gas and particles, through the relative velocity of the gas $V'^{\pm}_{\sim i} = V'^{\pm}_{gi} - V'^{\pm}_{pi}$ and use for this the momentum equation for the "gas + particles" system [2]

$$V'^{\pm}_{gi} = \frac{V'^{\pm}_{g0i} + \varkappa V'^{\pm}_{p0i} + \varkappa V'^{\pm}_{\sim i}}{1+\varkappa}, \tag{2}$$

$$V'^{\pm}_{pi} = \frac{V'^{\pm}_{g0i} + \varkappa V'^{\pm}_{p0i} - V'^{\pm}_{\sim i}}{1+\varkappa}. \tag{3}$$

Inserting expressions (2) and (3) into Eq. (1) yields, after integration,

$$\ln\frac{|V'^{\pm}_{\sim i} - g_i/N|}{|V'^{\pm}_{\sim 0i} - g_i/N|} = -N(1+\varkappa)t_p, \tag{4}$$

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TABLE 1. Shearing Stresses in a Gas Carrying Solid Particles, with and without Their Weight Taken into Account

$\varkappa_m$	4	2	4	4	4	4	4	
u_m, m/sec	20	20	20	20	20	20	20	10
D_p, μm	100	100	40	60	100	100	60	60
$10\text{x}\delta_u$, m	1,28	1,28	1,28	1,28	1,28	0,64	0,64	1,28
$10\text{x}D_p/\delta_u$	0,78	0,78	0,31	0,47	0,78	1,56	0,94	0,47
$u'_{g0}/V_f = k$	0,8	0,8	5,0	2,2	0,4	0,8	2,2	1,1
τ_g^-/τ_0	0,294	0,553	0,079	0,136	0,179	0,359	0,232	0,082
τ_g/τ_0	0,387	0,642	0,079	0,146	0,313	0,502	0,242	0,103
τ_p^-/τ_0	0,076	0,045	0,184	0,142	0,119	0,049	0,095	0,184
τ_p/τ_0	0,047	0,024	0,184	0,136	0,054	0,027	0,090	0,162
τ_m^-/τ_0	0,371	0,597	0,263	0,279	0,298	0,444	0,328	0,266
τ_m/τ_0	0,434	0,666	0,263	0,282	0,366	0,529	0,0332	0,256
τ_g/τ_g'	1,31	1,16	1,00	1,07	1,75	1,27	1,04	1,26
τ_m/τ_m^-	1,16	1,11	1,00	1,01	1,23	1,19	1,01	1,00

where $N = 18\mu_g/\rho_s D_p^2$, and t_p is the length of time in which particles interact with one mole of gas [2]

$$t_p = 2l_u/(V'^{\pm}_{p0i} + V'^{\pm}_{pi}). \tag{5}$$

Here l_u is the mixing length. After inserting expression (5) into expression (4) and a few simple transformations, we obtain an equation for $V'^{\pm}_{\sim i}$:

$$\ln\frac{|V'^{\pm}_{\sim i} - g_i/N|}{|V'^{\pm}_{\sim 0i} - g_i/N|} = \frac{2N\beta\delta_u(1+\varkappa)^2}{|V'^{\pm}_{g0i} + (1+2\varkappa)V'^{\pm}_{p0i} - V'^{\pm}_{\sim i}|}. \tag{6}$$

It is obvious that the magnitude of the fluctuation velocity of the gas at the end of the mole "life" will be significantly affected by the particle weight when their free-fall velocity is comparable in magnitude with the corresponding component of the fluctuation velocity of the gas.

It is well known that, if the drag coefficient C_x for a particle obeys Stokes' law, the relation between the gas velocity components V'_{g0i} and the corresponding components of the particle free-fall velocity V_{fi} is

$$\frac{V'_{g0i}}{V_{fi}} = \frac{18\mu_g V'_{g0i}}{\rho_s D_p^2 g_i}. \tag{7}$$

The right-hand side of relation (7) can be transformed to

$$\frac{V'_{g0i}}{V_{fi}} = \frac{18\mathrm{Fr}_i}{\mathrm{Re}}\,\frac{\rho_g}{\rho_s}. \tag{8}$$

where Fr_i is the Froude number and Re is the Reynolds number

$$\mathrm{Fr}_i = V'^2_{g0i}/g_i D_p,\ \ \mathrm{Re} = \rho_g V'_{g0i} D_p/\mu_g. \tag{9}$$

A dimensional analysis of relation (6) indicates that the relative fluctuation velocity of the gas at the end of the mole "life" must also depend on the admixture concentration $\varkappa$ and the relative particle size D_p/δ_u.

Shearing stresses in a gas, in a "gas" of particles, and in a mixture of gas with solid particles were calculated, as shown in Table 1, with and without taking into account the effect of the particle weight on the magnitude of the longitudinal components of the fluctuation velocities, gas and particles, in a vertical jet. These calculations are based on the assumption of an equilibrium flow in the average motion, i.e., with the velocity of the gas and particle velocity being equal and, furthermore, with a Schlichting distribution of both velocities as well as of the admixture concentration over the jet cross section

$$\frac{u}{u_m} = (1-\eta_u^{3/2})^2,\ \ \frac{\varkappa}{\varkappa_m} = (1-\eta_\varkappa^{3/2})^2. \tag{10}$$

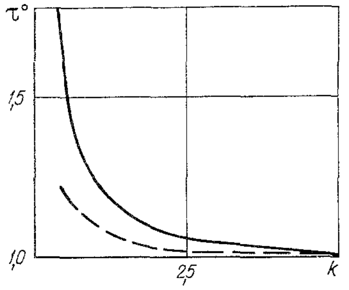

Fig. 1

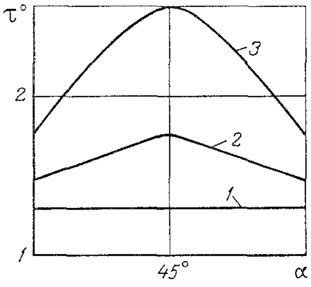

Fig. 2

Fig. 1. Dependence of the ratio of shearing stresses in the gas of a two-phase jet, respectively with and without the particle weight taken into account, on the parameter $k = u'_{g0}/v_f$ (solid line); this dependence for the mixture of gas with particles (dash line).

Fig. 2. Dependence of the ratio of shearing stresses in the gas, respectively with and without the particle weight taken into account, on the jet inclination angle: 1) k = 0.8; 2) 0.6; 3) 0.2.

As in an earlier study [3], the ratio $\delta_u/\delta_\varkappa$ of the ordinates of the jet boundaries in terms of velocity and concentration respectively was taken as equal to 1.5 and also to the mean (over the cross section) value of the Schmidt number. The shearing stresses in the gas and in the "gas" of particles were determined according to the relations

$$\tau_g = -\rho_g \langle u'_g v'_g \rangle, \quad \tau_p = -\rho_g \varkappa \langle u'_p v'_p \rangle . \tag{11}$$

The rms values of V'_{gi} and V'_{pi} were calculated here just as another earlier study [2], namely

$$V'_{gi} = (V'^{+}_{gi} - V'^{-}_{gi})/2, \quad V'_{pi} = (V'^{+}_{pi} - V'^{-}_{pi})/2, \tag{12}$$

and the shearing stresses in the mixture as the sum of those in the gas and those in the "gas" of particles. The calculations were made for an air jet with a dynamic viscosity $u_g = 0.18 \cdot 10^{-5}$ kg·sec/m^2 carrying spherical bronze particles with a density $\rho_s = 800$ kg/m^3 at $\eta_u = 0.2$ (dimensionless velocity ordinate). The empirical constant was taken as equal to 0.09 here. In the table τ_m and τ_0 denote shearing stresses in the gas mixture with solid particles and in the pure gas, respectively. The superscript " – " indicates that the given quantity has been calculated without taking the weight of particles into account.

According to the data in Table 1, the ratio $(V'_{g0}/V_f)_i$ of fluctuation velocity to free-fall velocity is the parameter which determines the effect of the weight of the admixture on the turbulence structure of the jet.

The solid curve in Fig. 1 depicts the dependence of the ratio of shearing stresses $\tau_g{}^0$, respectively with and without the effect of the particle weight taken into account, on the parameter u'_{g0}/v_f for a vertical jet with $\varkappa = 4$. The dash curve on the same diagram depicts the ratio of the respective shearing stresses $\tau_m{}^0$ in the mixture. Both curves and the data in Table 1 indicate that the effect of the particle weight on the magnitude of shearing stresses in the gas and in the mixture of gas with particles becomes stronger as the ratio k decreases and very appreciable when $k \leq 1$. At a certain value of this ratio the effect of gravity increases with increasing admixture concentration and with decreasing relative particle size. The graph in Fig. 2 depicts the dependence of the ratio of shearing stresses in the gas, respectively with and without the effect of the particle weight taken into account, on the jet inclination angle for three different ratios (0.8, 0.6, 0.2) of fluctuation velocity of the gas to resultant free-fall velocity. This graph indicates that shearing stresses in the gas can depend strongly on the inclination angle of a jet carrying a heavy admixture. This dependence becomes stronger as the ratio of fluctuation velocity of the gas to free-fall velocity of particles decreases.

Inasmuch as the magnitudes of the fluctuation velocities u'_{g0} and v'_{g0} vary over the jet cross sections and along the jet axis, so will also vary the ratio of fluctuation velocity to free-fall velocity of the particles. This means the effect of the particle weight on the fluctuation characteristics of a given jet will be different in different segments of it and can in some jet segment be appreciable. Therefore, if a preliminary estimate of the parameter k indicates that in any jet segments this ratio is close to or smaller than unity, the characteristics of a jet must be calculated with the effect of gravity on its turbulence structure taken into account.

NOTATION

C_x, drag coefficient for a particle; D_p, particle diameter; g_i, components of the acceleration g due to gravity acting on a particle in the direction of jet flow (g_i = g sin α) and in the direction normal to it (g_i = g-cos α); $V'^{\pm}_{p0i}$, $V'^{\pm}_{g0i}$, fluctuation components of the velocities of the particles and gas, respectively, at the end of a mole formation; V_{fi}, free-fall velocity of a particle; l_u, mixing length; m_p, particle mass; t_p, length of time of particle–mole interaction; $V'^{\pm}_{pi}$, $V'^{\pm}_{gi}$, positive and negative fluctuation velocities of particles and of the gas respectively, with the components $u'^{\pm}_p$, $u'^{\pm}_g$, $v'^{\pm}_p$, $v'^{\pm}_g$, $k = V'_{g0i}/V_{fi}$; $V'^{\pm}_{\sim i}$, relative velocity of the gas; α, jet inclination angle relative to the earth's surface; β, empirical constant; δ_u, $\delta_\varkappa$, jet boundaries in terms of velocity and concentration, respectively; $\eta_u = y/\delta_u$, dimensionless velocity ordinate; $\eta_\varkappa = y/\delta_\varkappa$, dimensionless concentration ordinate; $\varkappa$, admixture concentration; u_m, $\varkappa_m$, velocity and the concentration of the admixture at the jet axis, respectively; μ_g, dynamic viscosity of the gas; ρ_s, ρ_g, densities of the particle material and of the gas, respectively; τ_g, τ_p, shearing stresses in the gas and in the "gas" of particles, respectively; and τ_m, τ_0, shearing stresses in the mixture and in pure gas, respectively.

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NUMERICAL SOLUTION OF THE FORWARD ONE-DIMENSIONAL PROBLEM OF CRITICAL FLOW IN NOZZLES

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The principles of an algorithm are formulated for a numerical solution of problems of one-dimensional flow in nozzles with passage through a singularity. The results of calculations are compared with experimental data.

In engineering practice one often encounters problems involving the calculation of flow parameters for channels of variable cross section. In the simplest case this would be the flow of an ideal gas without friction and heat transfer. More complex problems include those involving the flow of a real gas with friction at the walls and with heat transfer, with expansion of two-phase or multiphase media accompanied by interphase interactions, with motion of multicomponent mixtures accompanied by chemical reactions, etc.

According to an earlier study [1], all these problems can be classified into forward and reverse ones. The latter are widely encountered in numerical analysis of the motion of various media through nozzles, inasmuch as here the solution does not have a singularity. In the analysis of flow through channels of a given geometry (so-called forward problem) there arise difficulties due to the fact that the solution contains a singularity of the saddle kind. It is not possible to obtain a continuous solution for critical flow and, therefore, special methods are used allowing the singularity to be taken out. Such methods are replacement of the steady-state problem with the transient one (method of stabilization), replacement of the forward problem in the vicinity of the critical point with the reverse one, and other methods. Such approaches have already been thoroughly surveyed [1, 2].

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